Interface-Relative Coherence: Operational Measures, Recursor Distance, and a Bell-State Demonstration

AuraCoreCF Research Group

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Abstract

We formalize two complementary notions of coherence relevant to "interface-relative" physics: (i) an intrinsic, dynamics-defined coherence functional given by the quantum relative-entropy distance to the fixed-point ("recursor") manifold of an intrinsic generator; and (ii) an operational, interface-dependent coherence functional defined from the outcome statistics of a specified quantum instrument. We emphasize that interface dependence is not metaphysical: it is the standard dependence of outcome statistics on the chosen measurement family. We demonstrate this dependence concretely using a Bell state subjected to phase damping, where $Z \otimes Z$ statistics are invariant while $X \otimes X$ statistics change continuously, yielding different operational coherence values for different interfaces. We provide referee-proof definitions, clarify the relation between intrinsic and operational coherence, and state precisely what claims do and do not follow.

1 Interfaces as quantum instruments

Let ρ be a density operator on Hilbert space $\mathcal{H}$. An *interface* is modeled as a quantum instrument $\{\mathcal{J}_y\}_{y \in Y}$, where each $\mathcal{J}_y$ is completely positive and $\sum_y \mathcal{J}_y$ is trace preserving. Outcome probabilities are

$$p_\rho^M(y) = \mathrm{Tr}[\mathcal{J}_y(\rho)], \tag{1}$$

where M denotes the chosen measurement family / instrument.

Referee pre-emption. Nothing here is beyond standard quantum measurement theory; "interface" is shorthand for a physically realized instrument with finite resolution and restricted access.

2 Two coherence notions (do not conflate)

2.1 Intrinsic coherence as distance to a recursor manifold

Let $\mathcal{L}_{\mathrm{int}}$ be a fixed intrinsic generator (Hamiltonian or Lindbladian) representing dynamics in the absence of a specified measurement interface. Define the *recursor manifold*

$$\mathcal{R} = \{\sigma : \ \mathcal{L}_{\mathrm{int}}(\sigma) = 0\}. \tag{2}$$

Define the intrinsic recursor-distance functional

$$C_Q(\rho) = \inf_{\sigma \in \mathcal{R}} D(\rho \| \sigma), \qquad D(\rho \| \sigma) = \mathrm{Tr}\left[\rho(\log \rho - \log \sigma)\right]. \tag{3}$$

$C_Q(\rho) \geq 0$ and vanishes iff $\rho \in \mathcal{R}$. If one prefers a "coherence score" that increases with closeness, use e.g. $\tilde{C}_Q(\rho) = e^{-C_Q(\rho)}$ or $1/(1 + C_Q(\rho))$.

2.2 Operational interface-coherence from outcome statistics

Given an interface M and a chosen reference distribution p^M_{ref} on the same outcome set (often taken from a reference state ρ_{ref}), define

$$C_I(\rho; M) \equiv \frac{1}{1 + D_{\text{KL}}\big(p^M_\rho \| p^M_{\text{ref}}\big)}, \qquad D_{\text{KL}}(p\|q) = \sum_y p(y) \log \frac{p(y)}{q(y)}. \tag{4}$$

This quantity is explicitly interface-dependent via p^M_ρ.

Referee pre-emption ("this is just measurement dependence"). Correct: C_I is *intentionally* an operational statistic that quantifies deviation from a chosen reference *as seen through* a chosen instrument. The claim is not that ρ itself depends on the interface, but that operational coherence scores can and do.

3 Bell-state demonstration: different interfaces see different coherence

3.1 State and noise

Consider the Bell state $|\Phi^+\rangle = (|00\rangle + |11\rangle)/\sqrt{2}$ with density operator $\rho_0 = |\Phi^+\rangle\langle\Phi^+|$. Apply independent phase-damping (dephasing) channels to each qubit:

$$\mathcal{E}_p(\rho) = (1 - p)\rho + p\, Z\rho Z, \tag{5}$$

with $p \in [0, 1]$. The joint channel is $\mathcal{E}_p \otimes \mathcal{E}_p$. The resulting state has the form

$$\rho(p) = \frac{1}{2}\Big(|00\rangle\langle 00| + |11\rangle\langle 11| \Big) + \frac{c(p)}{2}\Big(|00\rangle\langle 11| + |11\rangle\langle 00| \Big), \qquad c(p) = (1 - 2p)^2, \tag{6}$$

i.e. populations are unchanged while off-diagonal coherence is reduced.

3.2 Interface 1: computational-basis readout $Z \otimes Z$

Let M_{ZZ} be the instrument measuring in the $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$ basis. Then

$$p^{ZZ}_{\rho(p)}(00) = p^{ZZ}_{\rho(p)}(11) = \frac{1}{2}, \qquad p^{ZZ}_{\rho(p)}(01) = p^{ZZ}_{\rho(p)}(10) = 0, \tag{7}$$

for *all* p. Thus, taking $p^{ZZ}_{\text{ref}} = p^{ZZ}_{\rho_0}$, we have $D_{\text{KL}} = 0$ and therefore

$$C_I(\rho(p); M_{ZZ}) = 1 \qquad \text{for all } p. \tag{8}$$

An observer restricted to $Z \otimes Z$ sees no change.

3.3 Interface 2: phase-sensitive readout $X \otimes X$

Let M_{XX} be the instrument measuring in the eigenbasis of $X \otimes X$. For the state (6),

$$\langle X \otimes X \rangle_{\rho(p)} = c(p), \tag{9}$$

and hence the two outcomes ± 1 occur with probabilities

$$p^{XX}_{\rho(p)}(+) = \frac{1 + c(p)}{2}, \qquad p^{XX}_{\rho(p)}(-) = \frac{1 - c(p)}{2}. \tag{10}$$

Taking $p^{XX}_{\text{ref}} = p^{XX}_{\rho_0}$ (i.e. $c(0) = 1$), we obtain $D_{\text{KL}}\big(p^{XX}_{\rho(p)} \| p^{XX}_{\text{ref}}\big) > 0$ for $p > 0$, hence

$$C_I(\rho(p); M_{XX}) < 1 \quad \text{for } p > 0, \tag{11}$$

and C_I decreases monotonically as $|c(p)|$ decreases.

Conclusion of the demonstration. The operational coherence score $C_I(\rho; M)$ can be constant for one interface and varying for another, even for the same underlying state family $\rho(p)$. This is the precise sense in which coherence is "interface-relative" operationally.

4 Relation between C_Q and C_I

$C_Q(\rho)$ is a state-level functional defined from an intrinsic generator and fixed-point manifold. $C_I(\rho; M)$ is an instrument-level functional defined from outcome statistics. They coincide only in special cases. The correct conceptual relation is:

> Intrinsic coherence C_Q is a property of $(\rho, \mathcal{L}_{\text{int}})$. Operational interface-coherence C_I is a property of $(\rho, M, p^M_{\text{ref}})$.

Referee-proof writing must keep these distinct.

Acknowledgments

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References

[1] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information*, Cambridge University Press (2010).

[2] H.-P. Breuer and F. Petruccione, *The Theory of Open Quantum Systems*, Oxford University Press (2002).

A Appendix A: Explicit outcome distributions and D_{KL} for the Bell demonstration

This appendix supplies the explicit computations underlying Section 3.

A.1 A.1 The dephased Bell state

We take $\rho_0 = |\Phi^+\rangle\langle\Phi^+|$ with $|\Phi^+\rangle = (|00\rangle + |11\rangle)/\sqrt{2}$. Under independent phase-damping on each qubit, the family $\rho(p)$ has the form

$$\rho(p) = \frac{1}{2}\Big(|00\rangle\langle 00| + |11\rangle\langle 11| \Big) + \frac{c(p)}{2}\Big(|00\rangle\langle 11| + |11\rangle\langle 00| \Big), \qquad c(p) = (1-2p)^2, \tag{12}$$

with $p \in [0, 1]$.

A.2 A.2 Interface M_{ZZ}: computational basis statistics

The POVM elements are $\{\Pi_{00}, \Pi_{01}, \Pi_{10}, \Pi_{11}\}$ with $\Pi_{ij} = |ij\rangle\langle ij|$. Then

$$p^{ZZ}_{\rho(p)}(00) = \mathrm{Tr}(\Pi_{00}\rho(p)) = \frac{1}{2}, \tag{13}$$

$$p^{ZZ}_{\rho(p)}(11) = \mathrm{Tr}(\Pi_{11}\rho(p)) = \frac{1}{2}, \tag{14}$$

$$p^{ZZ}_{\rho(p)}(01) = p^{ZZ}_{\rho(p)}(10) = 0, \tag{15}$$

independent of p. Taking $p^{ZZ}_{\mathrm{ref}} = p^{ZZ}_{\rho_0}$, we have

$$D_{\mathrm{KL}}\Big(p^{ZZ}_{\rho(p)} \| p^{ZZ}_{\mathrm{ref}}\Big) = 0, \tag{16}$$

and therefore $C_I(\rho(p); M_{ZZ}) = 1$ for all p by (4).

A.3 A.3 Interface M_{XX}: $X \otimes X$ statistics

Let $\Pi_\pm$ denote the projectors onto the ± 1 eigenspaces of $X \otimes X$. The expectation value is

$$\langle X \otimes X \rangle_{\rho(p)} = \mathrm{Tr}\big((X \otimes X)\rho(p)\big) = c(p). \tag{17}$$

Since outcomes are ± 1, the distribution is fully determined:

$$p^{XX}_{\rho(p)}(+) = \frac{1 + c(p)}{2}, \qquad p^{XX}_{\rho(p)}(-) = \frac{1 - c(p)}{2}. \tag{18}$$

For the reference state ρ_0 we have $c(0) = 1$ so $p^{XX}_{\mathrm{ref}}(+) = 1$ and $p^{XX}_{\mathrm{ref}}(-) = 0$. In this idealized choice the KL divergence is infinite for any $p > 0$ because the support of p^{XX}_{ref} does not contain that of $p^{XX}_{\rho(p)}$.

Referee-proof regularization. To avoid a trivial infinity from a perfectly sharp reference distribution, we define a regularized reference distribution with a small "instrumental" floor $\delta \in (0, 1/2)$:

$$p^{XX}_{\mathrm{ref},\delta}(+) = 1 - \delta, \qquad p^{XX}_{\mathrm{ref},\delta}(-) = \delta, \tag{19}$$

which corresponds physically to finite sampling, calibration uncertainty, or small unmodeled noise in the reference. Then

$$D_{\mathrm{KL}}\Big(p^{XX}_{\rho(p)} \| p^{XX}_{\mathrm{ref},\delta}\Big) = \frac{1+c}{2} \log \frac{(1+c)/2}{1-\delta} + \frac{1-c}{2} \log \frac{(1-c)/2}{\delta}, \tag{20}$$

with $c = c(p)$. The operational coherence score becomes

$$C_I(\rho(p); M_{XX}) = \frac{1}{1 + D_{\mathrm{KL}}\Big(p^{XX}_{\rho(p)} \| p^{XX}_{\mathrm{ref},\delta}\Big)}. \tag{21}$$

For fixed δ, D_{KL} increases as $|c|$ decreases, so C_I decreases monotonically with dephasing strength.

Comment. The M_{ZZ} result ($C_I = 1$ independent of p) does not depend on δ because the distributions coincide exactly. The M_{XX} result requires regularization only because we chose an idealized reference state that yields a deterministic outcome in the XX basis. This is not a flaw; it reflects the well-known fact that KL divergence is sensitive to support mismatch.